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Abstract

Alchourrón's contributory conditionals and refinement of normative systems

In his last papers (e.g. Alchourrón, 1996), Alchourrón attacked non-monotonic logics as philosophically unsound for the representation of defeasible reasoning in general and in law. Alchourrón's convictions about the role of science and legal dogmatics as descriptive of general norms, led him to view defeasibility as *susceptibility to change*. Therefore the formal approach he considered philosophically sound to defeasibility regarded the qualification of general conditionals by means of an AGM revision operator on the antecedent of conditionals, obtaining what he called *contributory conditionals*, i.e. the explicit antecedent is a necessary condition of a sufficient condition for the conclusion.

He defined a defeasible conditional in the object language as $a>b=fa\Rightarrow b$. and interpreted the selection f as a revision operator (fa represents the joint assertion of a with its consistent implicit assumptions). He called this logic DFT, which he proved to be equivalent to Lewis's VTA.

Given that the set of assumptions must be consistent with the explicit contributory condition, we may interpret fa as a revision of a fixed set K of implicit assumptions by sentence a, i.e. $f\text{a}=*_{\text{K}}(\text{a})=\ \text{K}*a$. Alchourrón's efforts consisted in showing the logical equivalence of the dynamics of revision with non-monotonic logics, so that he could conclude, in the logical level: *we do not need snakes.* To illustrate this point he compared two procedures to analyze defeasibility:

(1) Birds (b) fly (a); (2) Penguins (p) do not fly ($\neg a$)

General (revised) Conditionals	Defeasible Conditionals
G1. $(b\wedge\neg p)\Rightarrow a$	d1. $b>a$
G2. $p\Rightarrow\neg a$	d2. $p>\neg a$

Both procedures avoid the inconsistent and counterintuitive conclusion that penguins, which are birds, fly and do not fly. Given a defeasible conditional where d2 is preferred to d1 we derive the correct conclusion that penguins do not fly. But in Alchourrón's formalization of defeasible conditionals d1 is $fb\Rightarrow a$ and d2 is $fp\Rightarrow\neg a$. Provided that the choice related to fb and fp is given by the equations $Ch[b]=[b\wedge\neg p]$ and $Ch[p]=[p]$ (so we have $fb\Leftrightarrow(b\wedge\neg p)$ and $fp\Leftrightarrow$p) the procedure of representation with defeasible conditionals is logically equivalent to the procedure that uses general (revised) conditionals.

Alternatively, (Maranhão, 2007) developed an operator *on the theory or normative system* by which conflicting sentences or conditionals are qualified (i.e. the antecedent of the conditional is extended), rather than deleted, The idea is very simple. Suppose one believes that water boils at 100^0C (a). Once an experiment to boil water in La Paz fails ($\neg a$) she realizes that the real mistake was to believe that water boils at 100^oC even if she is not at sea level ($b\rightarrow\neg a$, where b is the defeating condition). Therefore what she has to do is to exclude this weaker mistaken belief, preserving the other conditional according to which water boils at 100^0C at sea level ($\neg b\rightarrow a$). I called *internal refinement* the operator which restricts the original belief. The complementary

move which would consist in adding the defeating conditional ($b\rightarrow\neg a$) to the theory was called *global refinement*.

The construction of the internal refinement operator is based on AGM contraction functions. First define a conditioning function h on the set of formulas that will select the sentence representing the defeating condition. Then, assuming a contraction function $\div$ and a conditioning function h, define the internal refinement operator of a theory K by sentence a as a contraction by the defeating conditional, i.e. $K\#a= K\div h(a)\rightarrow a$. It turns out that, given some suitable restrictions on the conditioning function, the internal refinement function AGM-contraction functions satisfying the additional postulate $\neg h(a)\rightarrow a \in K\#a$ (*preservation postulate*) according to which the contracted sentence is preserved when the defeating condition is absent.

The function of global refinement of a theory K by a sentence a (notation $K\bullet a$) is constructed out of internal refinement with an obvious move: $K\bullet a = Cn(K\#a \cup \{\neg h(a)\rightarrow a\})$. Such operator satisfies the AGM revision postulates except for self-deducibility, which means that global refinement, although with obvious affinities, is not a revision.

A general procedure to avoid inconsistency with all the defeaters of a conditional within the theory is obtained by an internal refinement operation on a theory Th based on a proper selection by the conditioning function so that $h(b\Rightarrow a)= \vee\{d: d\Rightarrow\neg a \in Th\}$. The internal refinement function will then provide as output that $(b\wedge\{\neg d\})\Rightarrow a \in Th\#\,(b\Rightarrow a)$.

The advantage of using an operator on the theory and not on the (antecedent of a) conditional is that we have an account of the whole resulting theory after the revision procedure, by which we capture defeasibility.

Nevertheless the similarity of refinement and Alchourrón's conditionals is evident and the paper intends to explore possible translations, applied to DFT and also Alchourrón's logic of normative propositions. The key to translations is the conditioning function h. Contributory conditionals may be seen as the result of refinement operators where the conditioning function chooses the consistent implicit assumptions associated with the antecedent of the conditional, that is, the function h would satisfy the following restriction $h(a\Rightarrow b)= \neg(\wedge \mathrm{K}\div\neg a)$. On its turn, the consistent set of implicit assumptions associated with the antecedent of a conditional $a\Rightarrow b$ may be constructed so as to observe the conditioning function of a refinement operator. The idea is to use a selection function s in the construction of the contraction operator on the set K of implicit assumptions satisfying the following restriction: $\mathrm{s}(\mathrm{K}\perp\neg a)=\{\mathrm{X}\in \mathrm{K}\perp\neg a: \neg h(a\Rightarrow b)\in \mathrm{X}\}$.

Quoted papers:

Alchourrón, C. (1996) "Detachment and Defeasibility in Deontic Logic", *Studia Logica*, 57, pp. 5-18.

Maranhão, J.S.A. (2007) "Refining Beliefs" in J-Y Béziau, A. Costa-Leite (eds.), *Perspectives on Universal Logic*, 335-349, Polimetrica International Scientific Publisher, Monza/Italy.