Solving the Bottleneck Traveling Salesman Problem Using the Lin-Kernighan-Helsgaun Algorithm

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Abstract

The Bottleneck Traveling Salesman Problem (BTSP) is a variation of the Traveling Salesman Problem (TSP). The Bottleneck Traveling Salesman Problem asks for a tour in which the largest edge is as small as possible. It is well known that any BTSP instance can be solved using a TSP solver. This paper evaluates the performance of the state-of-the art TSP solver Lin-Kernighan-Helsgaun (LKH) on a large number of benchmark instances. When LKH was used as a black box, without any modifications, most instances with up to 115,475 vertices could be solved to optimality in a reasonable time. A small revision of LKH made it possible to solve BTSP instances with up to one million vertices. The program, named BLKH, is free of charge for academic and non-commercial use and can be downloaded in source code. The program is also applicable to the Maximum Scatter TSP (MSTSP), which asks for a tour in which the smallest edge is as large possible.

Keywords: Bottleneck traveling salesman problem, BTSP, Maximum scatter traveling salesman problem, MSTSP, Traveling salesman problem, TSP, Lin-Kernighan

Mathematics Subject Classification: 90C27, 90C35, 90C59

1. Introduction

The Bottleneck Traveling Salesman Problem (BTSP) is a variation of the Traveling Salesman Problem (TSP). The BTSP asks for a tour in which the largest edge is as small as possible. The problem has numerous applications, including transportation of perishable goods, work-force scheduling [1], and minimizing the makespan in a two-machine flowshop [2].

The BTSP is defined on a complete graph $G = (V, E)$, where $V=\{v_1....v_n\}$ is the vertex set and $E=\{(v_i,v_j) : v_i, v_j \in V, i \neq j\}$ is the edge set. A non-negative cost c_{ij} is associated with each edge (v_i, v_j). The BTSP can be stated as the problem of finding a Hamiltonian cycle whose largest edge is as small as possible. If the cost matrix $C = (c_{ij})$ is symmetric, i.e., $c_{ij} = c_{ji}$ for all $i, j, i{\neq}j$, the problem is called *symmetric*. Otherwise it is called *asymmetric*.

Just like the TSP, the BTSP is NP-hard [3].

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Figure 1 shows the optimal BTSP tour for usa1097, an instance consisting of 1097 cities in the adjoining 48 U.S. states, plus the District of Columbia. Its total length is 127,786,915 meters, and its bottleneck edge has a length of 175,360 meters. In comparison, the optimal TSP tour for this instance has a total length of 71,109,602 meters and a bottleneck edge length of 195,002 meters.

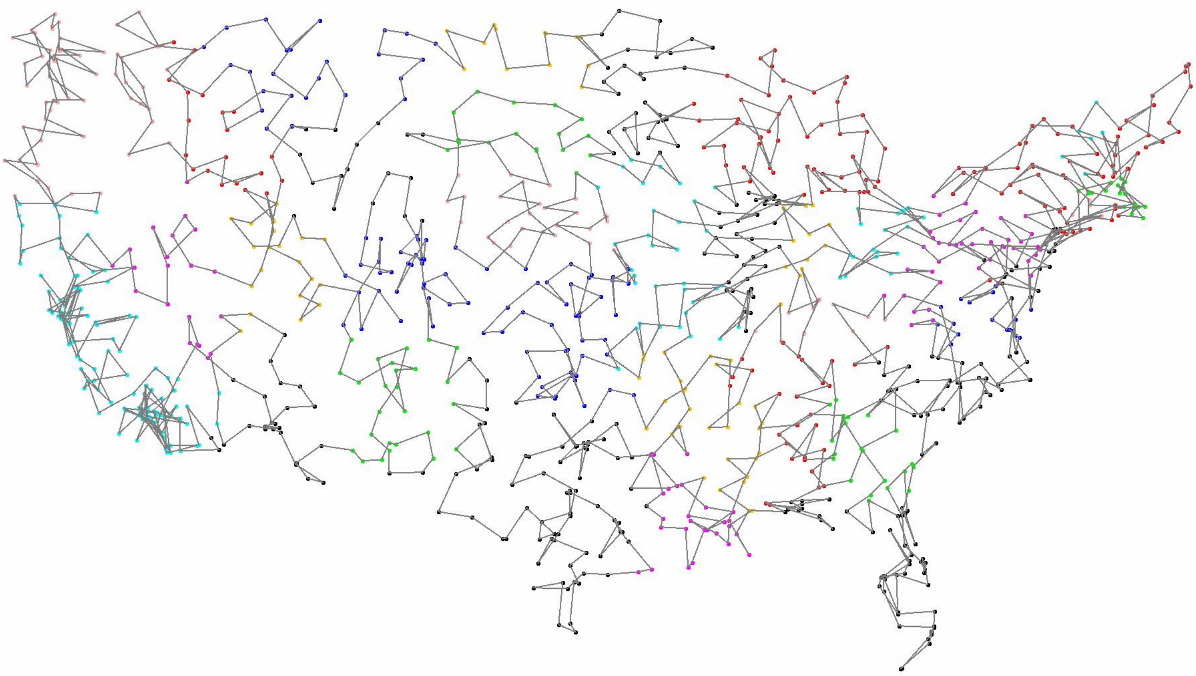

Figure 1 *Optimal BTSP tour for usa1097.*

It is well known that any BTSP instance can be solved using a TSP solver [3], either after transforming the BTSP instance into a standard TSP instance, or by solving the BTSP instance as a sequence of $O(\log n)$ standard TSP instances using Edmonds and Fulkerson's well known threshold algorithm for bottleneck problems [4]. The first approach, however, does not have much practical value since the edge costs of the transformed instance grows exponentially. For this reason, the second approach has been chosen.

LKH [5, 6] is a powerful local search heuristic for the TSP based on the variable depth local search of Lin and Kernighan [7]. Among its characteristics may be mentioned its use of 1-tree approximation for determining a candidate edge set, extension of the basic search step, and effective rules for directing and pruning the search. LKH is available free of charge for scientific and educational purposes from http://www.ruc.dk/~keld/research/LKH. The following section describes how LKH can be used as a black box to solve the BTSP.

2. Implementing a BTSP Solver Based on LKH

Input to LKH is given in two files:

(1) A *problem file* in TSPLIB format [8], which contains a specification of the TSP instance to be solved. A problem may be symmetric or asymmetric. In the latter case, the problem is transformed by LKH into a symmetric one with $2n$ vertices.
(2) A *parameter file*, which contains the name of the problem file, together with some parameter values that control the solution process. Parameters that are not specified in this file are given suitable default values.

The BTSP solver, named BLKH, starts by reading the parameter file as well as the problem file. A lower bound for the bottleneck cost is then computed and used for solving the BTSP. An outline of the main program (written in C) is given below.

```
int main(int argc, char *argv[]) {
    int Bottleneck, Bound, LowerBound;

    ReadParameters();
    ReadProblem();

    LowerBound = TwoMax();
    if ((Bound = BBSSP(LowerBound)) > LowerBound)
        LowerBound = Bound;
    if (ProblemType == ATSP) {
        if ((Bound = BBSSPA(LowerBound)) > LowerBound)
            LowerBound = Bound;
        if ((Bound = BSCSSP(LowerBound)) > LowerBound)
            LowerBound = Bound;
        if ((Bound = BAP(LowerBound)) > LowerBound)
            LowerBound = Bound;
    }

    Bottleneck = SolveBTSP(LowerBound);
}
```

Comments:

1. The algorithms for computing a lower bound are similar to those used by John LaRusic et al. [9, 10].

    - The TwoMax function computes the 2-Max bound.
    - The BBSSP function attempts to improve this bound by solving the Bottleneck Biconnected Spanning Subgraph Problem on a symmetric cost matrix. For asymmetric instances, the following symmetric relaxation of the asymmetric cost matrix is used: $c'_{ij} = \min\{c_{ij}, c_{ji}\}$.
    - The BBSSPA function attempts to improve the current lower bound for an asymmetric instance by solving the Bottleneck Biconnected Spanning Subgraph Problem on a $2n$x$2n$ symmetric cost matrix constructed from the asymmetric nxn cost matrix using Jonker and Volgenant's well known ATSP-to-TSP transformation [11].
    - The BSCSSP function attempts to improve the current lower bound for an asymmetric instance by solving the Bottleneck Strongly Connected Spanning Subgraph Problem.

- Finally, the BAP function attempts to improve the current lower bound for an asymmetric instance by solving the Bottleneck Assignment Problem.

See [10] for a more precise description of these bounds.

The implementation in BLKH of the lower bound algorithms for the last four bounds differs on three points from the implementation used in [10]:

(1) The current lower bound is used as an initial lower limit for the search interval.
(2) The binary search is based on interval midpoints, in contrast to [10], which uses binary search based on medians of edge costs.
(3) An initial test is made in order to decide whether it is possible to raise the current lower bound. If not, the binary search is skipped.

Computational evaluation has shown that BLKH's strategy is efficient in both runtime and memory usage.

The four binary search algorithms follow the same pattern. Below is shown the algorithm for the BBSSP function.

```
int BBSSP(int Low) {
    int High = INT_MAX, Mid;

    if (Biconnected(Low))
        return Low;
    Low++;
    while (Low < High) {
        Mid = Low + (High - Low) / 2;
        if (Biconnected(Mid))
            High = Mid;
        else
            Low = Mid + 1;
    }
    return Low;
}
```

3. After the lower bound has been computed, the SolveBTSP function is called in order to solve the given BTSP instance (using LKH). The function returns the bottleneck cost of the obtained tour.

The implementation of the SolveBTSP function is shown below.

```
int SolveBTSP(int LowerBound) {
    int Low = LowerBound, High = INT_MAX, Bottleneck;

    Bottleneck = High = SolveTransformedTSP(Low, High);
    while (Low < High && Bottleneck != LowerBound) {
        int B = SolveTransformedTSP(Low, High);
        if (B < Bottleneck) {
            Bottleneck = High = B;
            if (High <= Low)
                Low = LowerBound + (High - LowerBound) / 2;
        } else
            Low += (High - Low + 1) / 2;
    }
    return Bottleneck;
}
```

Comments:

1. As in the computation of the lower bound, binary search is also used here. The integers Low and High denote the current search interval for the bottleneck cost. The algorithm differs from the one described in [10] on the following points:

    - The binary search is based on interval midpoints, in contrast to [10], which uses binary search based on medians of edge costs.
    - LKH is used for solving the TSPs, whereas [10] uses Concorde's implementation of the Lin-Kernighan algorithm [12].
    - Initially, an upper bound for the bottleneck cost is computed by executing LKH. If this upper bound is equal to the lower bound, optimum has been found, and the binary search is skipped. This is advantageous, since the lower bound is often the true optimum.
    - The algorithm is simpler, since the 'shake' strategy of [9] and [10] is not used.

2. The SolveTransformedTSP function (1) transforms the original instance into a standard TSP instance according to the current values of Low and High, (2) solves this transformed instance using LKH, and (3) returns the bottleneck cost for the obtained tour. The TSP instances are constructed by the following transformation of the original cost matrix:

$$c'_{ij} = \begin{cases} 0 & if\ c_{ij} \leq Low \\ c_{ij} & if\ Low < c_{ij} < High \\ c_{ij} + \dfrac{INT\_MAX}{4} & if\ c_{ij} \geq High \end{cases}$$

The implementation of the SolveTransformedTSP function is shown below in pseudo code.

```
int SolveTransformedTSP(int Low, int High) {
    Create problem file;
    if (High == INT_MAX) {
        Create parameter file with special values;
        Create candidate file;
    } else
        Create parameter file based on the input parameter file;
    SolveTSP();
    return bottleneck cost of the obtained tour;
}
```

Comments:

1. First, the transformed cost matrix is written to a file that adheres to the TSPLIB file format [8]. A suitable parameter file is then created, and the standard TSP instance is solved by LKH.

2. When SolveTransformedTSP is called for the first time (High = INT_MAX), some special parameter values are chosen and a file that specifies candidate edges is created. The candidate edges are those edges that have a transformed cost of zero. However, since the number of such edges may be very large, pruning may be necessary in order to keep running time low. For symmetric instances, a maximum of 100 candidate edges are allowed to emanate from any vertex. If pruning for a vertex is necessary, its emanating candidate edges are chosen as the connections to its 100 nearest neighbor vertices. For asymmetric instances, the maximum number of allowable emanating candidate edges for any vertex is 1000.

   The special parameter values are:

   ```
   CANDIDATE_FILE = <name of candidate file>
   GAIN23 = NO
   MAX_CANDIDATES = 0
   MOVE_TYPE = 3
   OPTIMUM = 0
   RESTRICTED_SEARCH = NO
   SUBGRADIENT = NO
   SUBSEQUENT_MOVE_TYPE = 2
   ```

   These parameter settings cause LKH to use non-restricted 3-opt as its basic move type (`MOVE_TYPE = 3`, `RESTRICTED_SEARCH = NO`). Non-sequential moves are not explored (`Gain23 = NO`), and in any variable *k*-opt move, all submoves following the initial submove are 2-opt moves (`SUBSEQUENT_MOVE_TYPE = 2`). Only the edges in the candidate file will be used as candidate edges (`MAX_CANDIDATES = 0`), and no attempt is made to improve the lower bound of the TSP by subgradient optimization (`SUBGRADIENT = NO`). The setting `OPTIMUM = 0` causes LKH to stop as soon as tour cost of zero is found. Experiments have shown that these parameter settings work very well, both when it comes to running time and tour quality.

3. For all subsequent calls of SolveTransformedTSP, the parameter settings are taken from the input parameter file given to BLKH.

4. The generated TSP instance is solved by the SolveTSP function by executing LKH as a child process (using the Standard C Library function popen()).

3. Improving the BTSP Solver

The solver described in the previous section performs well on instances with up to 100,000 vertices. However, for large instances the produced problem files are big, and LKH's usage of RAM is high (it grows a n^2). For example, for the Euclidean 100,000-vertex instance E100k.0, the problem file occupies 35 GB, and LKH uses 22 GB of RAM for solving the TSP instances.

For geometric instances specified by coordinates, like E100k.0, such excessive usage of memory resources can be avoided by a minor revision of LKH. Two additional parameters, LOW and HIGH, are introduced, which allows LKH to create the desired set of candidate edges and compute transformed edge costs on the fly. This exempts BLKH from creating problem files and candidate files. Not only does this make possible solution of very large geometric instances, as seen in Table 1, running time is also reduced. The first column specifies the name of the instance, and the second the number of vertices. The C- and E-instances are clustered and uniformly distributed Euclidean random instances, respectively [13]. The instance usa115475 is taken from [14]. The last two columns give the total running times to find optima when the original version of LKH is used (BLKH_O) and when the revised version of LKH is used (BLKH_R). Running times are measured in seconds on an iMac 3.4 GHz Intel Core i7 with 32 GB RAM.

Name	Size	BLKH_O Total time	BLKH_R Total time
C1k.0	1,000	0.30	0.10
E1k.0	1,000	0.30	0.08
C3k.0	3,162	3.05	0.75
E3k.0	3,126	2.49	0.16
C10k.0	10,000	27.80	4.86
E10k.0	10,000	24.79	1.77
C31k.0	31,623	298.25	56.37
E31k.0	31,623	323.16	78.39
C100k.0	100,000	3,567.45	848.44
E100k.0	100,000	2,998.24	280.10
usa115475	115,475	3,865.90	404.64

Table 1 *Running times for BLKH using the two versions of LKH.*

4. Computational Evaluation

BLKH was coded in C and run under Linux on an iMac 3.4GHz Intel Core i7 with 32 GB RAM. The implementation is based on version 2.0.7 of LKH, which was revised as described in the previous section.

The performance of BLKH has been evaluated on 339 symmetric and 352 asymmetric benchmark instances. For all instances, the currently best known solutions were either found or improved.

4.1 Performance on Symmetric Instances

For all symmetric instances, the following parameter settings for BLKH were chosen:

> PROBLEM_FILE = TSP_INSTANCES/<*instance name*>.tsp
> INITIAL_PERIOD = 100
> MAX_TRIALS = 100
> MOVE_TYPE = 3
> OPTIMUM = <*best known cost*>
> RUNS = 1

An explanation is given below:

> PROBLEM_FILE: The symmetric test instances have been placed in the directory TSP_INSTANCES and have filename extension ".tsp".
>
> INITIAL_PERIOD: The candidate sets that are used in the Lin-Kernighan search process are found using a Held-Karp subgradient ascent algorithm based on minimum 1-trees [16]. The parameter specifies the length of the first period in the Held-Karp ascent (default is $n/2$).
>
> MAX_TRIALS: Maximum number of trials (iterations) to be performed by the iterated Lin-Kernighan procedure (default is n).
>
> MOVE_TYPE: Basic k-opt move type used in the Lin-Kernighan search (default is 5). LKH offers the possibility of using higher-order and/or non-sequential move types in order to improve the solution quality. However, preliminary tests showed that 3-opt moves were sufficient for solving the symmetric benchmark instances.
>
> OPTIMUM: This parameter may be used to supply a best known solution cost. The algorithm will stop if this value is reached during the search process.
>
> RUNS: Number of runs to be performed by LKH. Set to 1, since preliminary tests showed that optimum could be found for all benchmark instances using only one run (default is 10).

Tables 2-9 show the test results for the same symmetric benchmark instances as were used by John LaRusic et al. in [9]. Each test was repeated ten times. The column headers are as follows:

Name: the instance name.

Size: the number of vertices.

Lower bound: The lower bound (calculated by TwoMax and BBSSP).

Success: The number of tests in which the best value was found.

BLKH Best value: The best value found by BLKH.

BLKH Total time: The average CPU time, in seconds, used by BLKH for one test (includes the time for calculating the lower bound).

JLR Best value: The best value found John LaRusic et al.'s implementation (JLR).

JLR Total time: The average CPU time, in seconds, used by JLR for one test (includes the time for calculating the lower bound).

As can be seen from the test results in Tables 2-9, BLKH and JLR find the same best values for these instances, but BLKH uses much less CPU time than JLR, even when it is taken into account that BLKH was run on a more powerful processor (3.4 GHz Intel i7) than the processor used for running JLR (2.8 GHz Intel Xeon). That BLKH finds solutions much faster than JLR is particularly obvious from the test results for the specially constructed hard instances (Tables 8-9).

Due to memory limitations, JLR was not able to solve instances with more than 31,623 vertices. This is not an obstacle for BLKH when using the slightly revised version of LKH. Table 10 gives the computational results for 30 instances with between 31,623 and 1,000,000 vertices. The column "Lower bound time" gives the time, in seconds, used to find the lower bound. All runs required less than 1 GB of RAM. As can bee seen, optimum solutions were found for all instances.

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
a280	280	20	10/10	20	0.01	20	0.92
ali535	535	3,889	10/10	3,889	0.07	3,889	7.55
att48	48	519	10/10	519	0.00	519	0.08
att532	532	229	10/10	229	0.05	229	3.85
bayg29	29	111	10/10	111	0.00	111	0.03
bays29	29	154	10/10	154	0.00	154	0.03
berlin52	52	475	10/10	475	0.00	475	0.13
bier127	127	7,486	10/10	7,486	0.00	7,486	0.78
brazil58	58	2,149	10/10	2,149	0.00	2,149	0.18
brd14051	14,051	1,306	10/10	1,306	10.42	1,306	187.73
brg180	180	30	10/10	30	0.01	30	1.00
burma14	14	418	10/10	418	0.00	418	0.01
ch130	130	142	10/10	142	0.00	142	0.42
ch150	150	93	10/10	93	0.00	93	0.38
d1291	1,291	1,289	10/10	1,289	0.04	1,289	12.49
d15112	15,112	1,370	10/10	1,370	2.80	1,370	227.86
d1655	1,655	1,476	10/10	1,476	0.07	1,476	13.44
d18512	18,512	476	10/10	476	6.70	476	304.16
d198	198	1,380	10/10	1,380	0.00	1,380	1.42
d2103	2,103	1,133	10/10	1,133	0.08	1,133	14.65
d493	493	2,008	10/10	2,008	0.01	2,008	5.34
d657	657	1,368	10/10	1,368	0.01	1,368	6.92
dantzig42	42	35	10/10	35	0.00	35	0.05
dsj1000	1,000	295,939	10/10	295,939	0.08	295,939	11.96
eil101	101	13	10/10	13	0.00	13	0.25
eil51	51	13	10/10	13	0.00	13	0.05
eil76	76	16	10/10	16	0.00	16	0.22
fl1400	1,400	530	10/10	530	0.05	530	10.58
fl1577	1,577	431	10/10	431	0.11	431	10.94
fl3795	3,795	528	10/10	528	0.27	528	22.01
fl417	417	472	10/10	472	0.01	472	5.67
fnl4461	4,461	132	10/10	132	0.42	132	23.59
fri26	26	93	10/10	93	0.00	93	0.03
gil262	262	23	10/10	23	0.00	23	0.96
gr120	120	220	10/10	220	0.00	220	0.49
gr137	137	2,132	10/10	2,132	0.01	2,132	0.69
gr17	17	282	10/10	282	0.00	282	0.01
gr202	202	2,230	10/10	2,230	0.01	2,230	1.97
gr21	21	355	10/10	355	0.00	355	0.02
gr229	229	4,027	10/10	4,027	0.01	4,027	1.85
gr24	24	108	10/10	108	0.00	108	0.02
gr431	431	4,027	10/10	4,027	0.03	4,027	5.46
gr48	48	227	10/10	227	0.00	227	0.07
gr666	666	4,264	10/10	4,264	0.06	4,264	10.33
gr96	96	2,807	10/10	2,807	0.00	2,807	0.43
hk48	48	534	10/10	534	0.00	534	0.07
kroA100	100	475	10/10	475	0.00	475	0.20
kroA150	150	392	10/10	392	0.00	392	0.34
kroA200	200	408	10/10	408	0.00	408	0.68
kroB100	100	530	10/10	530	0.00	530	0.25
kroB150	150	436	10/10	436	0.00	436	0.40
kroB200	200	344	10/10	344	0.01	344	0.55
kroC100	100	498	10/10	498	0.00	498	0.19
kroD100	100	491	10/10	491	0.00	491	0.19

Table 2 *Results for symmetric TSPLIB instances [8] (Part I).*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total Time
kroE100	100	490	10/10	490	0.00	490	0.19
lin105	105	487	10/10	487	0.00	487	0.28
lin318	318	487	10/10	487	0.01	487	1.41
nrw1379	1,379	105	10/10	105	0.04	105	8.44
p654	654	1,223	10/10	1,223	0.04	1,223	5.40
pa561	561	16	10/10	16	0.03	16	3.44
pcb1173	1,173	243	10/10	243	0.03	243	8.30
pcb3038	3,038	198	10/10	198	0.42	198	13.58
pcb442	442	500	10/10	500	0.01	500	2.94
pla7397	7,397	81,438	10/10	81,438	5.52	81,438	181.79
pr1002	1,002	2,129	10/10	2,129	0.05	2,129	7.07
pr107	107	7,050	10/10	7,050	0.00	7,050	0.42
pr124	124	3,302	10/10	3,302	0.00	3,302	0.48
pr136	136	2,976	10/10	2,976	0.00	2,976	0.54
pr144	144	2,570	10/10	2,570	0.00	2,570	0.51
pr152	152	5,553	10/10	5,553	0.00	5,553	0.67
pr226	226	3,250	10/10	3,250	0.00	3,250	0.94
pr2392	2,392	481	10/10	481	0.49	481	12.03
pr264	264	4,701	10/10	4,701	0.01	4,701	1.83
pr299	299	498	10/10	498	0.02	498	2.41
pr439	439	2,384	10/10	2,384	0.00	2,384	3.07
pr76	76	3,946	10/10	3,946	0.00	3,946	0.20
rat195	195	21	10/10	21	0.00	21	0.51
rat575	575	23	10/10	23	0.01	23	2.82
rat783	783	26	10/10	26	0.01	26	5.46
rat99	99	20	10/10	20	0.00	20	0.10
rd100	100	221	10/10	221	0.00	221	0.30
rd400	400	104	10/10	104	0.01	104	1.94
rl11849	11,849	842	10/10	842	5.53	842	149.59
rl1304	1,304	1,535	10/10	1,535	0.07	1,535	9.35
rl1323	1,323	2,489	10/10	2,489	0.08	2,489	10.87
rl1889	1,889	896	10/10	896	0.18	896	9.72
rl5915	5,915	602	10/10	602	1.67	602	229.82
rl5934	5,934	896	10/10	896	1.55	896	60.29
si1032	1,032	362	10/10	362	0.15	362	21.84
si175	175	177	10/10	177	0.01	177	0.68
si535	535	227	10/10	227	0.04	227	4.35
st70	70	24	10/10	24	0.00	24	0.15
swiss42	42	67	10/10	67	0.00	67	0.06
ts225	500	1,000	10/10	1,000	0.06	1,000	2.99
tsp225	225	36	10/10	36	0.00	36	0.75
u1060	1,060	2,378	10/10	2,378	0.02	2,378	8.39
u1432	1,432	300	10/10	300	0.04	300	7.21
u159	159	800	10/10	800	0.00	800	0.52
u1817	1,817	234	10/10	234	0.06	234	8.61
u2152	2,152	105	10/10	105	0.09	105	25.66
u2319	2,319	224	10/10	224	0.09	224	10.03
u574	574	345	10/10	345	0.01	345	3.73
u724	724	170	10/10	170	0.02	170	4.00
ulysses16	16	1,504	10/10	1,504	0.00	1,504	0.01
ulysses22	22	1,504	10/10	1,504	0.00	1,504	0.03
usa13509	13,509	16,754	10/10	16,754	3.08	16,754	211.14
vm1084	1,084	998	10/10	998	0.04	998	6.91
vm1748	1,748	1,017	10/10	1,017	0.05	1,017	9.47

Table 3 *Results for symmetric TSPLIB instances [8] (Part II).*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
C10k.0	10,000	161,062	10/10	161,062	4.86	161,062	225.32
C10k.1	10,000	94,139	10/10	94,139	5.22	94,139	178.63
C10k.2	10,000	121,209	10/10	121,209	4.47	121,209	173.08
C1k.0	1,000	290,552	10/10	290,552	0.10	290,552	13.44
C1k.1	1,000	335,184	10/10	335,184	0.06	335,184	12.12
C1k.2	1,000	225,295	10/10	225,295	0.07	225,295	10.18
C1k.3	1,000	416,768	10/10	416,768	0.07	416,768	27.66
C1k.4	1,000	318,930	10/10	318,930	0.09	318,930	10.86
C1k.5	1,000	260,389	10/10	260,389	0.06	260,389	30.25
C1k.6	1,000	175,740	10/10	175,740	0.07	175,740	10.94
C1k.7	1,000	301,366	10/10	301,366	0.06	301,366	12.28
C1k.8	1,000	246,519	10/10	246,519	0.07	246,519	10.75
C1k.9	1,000	208,091	10/10	208,091	0.07	208,091	10.22
C31k.0	31,623	84,302	10/10	84,302	56.37	84,302	1,680.74
C3k.0	3,162	252,245	10/10	252,245	0.75	252,245	32.21
C3k.1	3,162	167,466	10/10	167,466	0.46	167,466	74.67
C3k.2	3,162	194,007	10/10	194,007	0.68	194,007	27.43
C3k.3	3,162	180,852	10/10	180,852	0.61	180,852	41.94
C3k.4	3,162	180,583	10/10	180,583	0.52	180,583	29.63
E10k.0	10,000	20,174	10/10	20,174	1.77	20,174	119.03
E10k.1	10,000	22,883	10/10	22,883	5.85	22,883	121.10
E10k.2	10,000	20,208	10/10	20,208	7.74	20,208	119.04
E1k.0	1,000	64,739	10/10	64,739	0.08	64,739	6.09
E1k.1	1,000	67,476	10/10	67,476	0.02	67,476	6.50
E1k.2	1,000	88,522	10/10	88,522	0.06	88,522	8.08
E1k.3	1,000	59,220	10/10	59,220	0.02	59,220	5.73
E1k.4	1,000	68,259	10/10	68,259	0.02	68,259	6.54
E1k.5	1,000	61,406	10/10	61,406	0.02	61,406	5.85
E1k.6	1,000	68,777	10/10	68,777	0.02	68,777	6.70
E1k.7	1,000	70,389	10/10	70,389	0.02	70,389	6.77
E1k.8	1,000	57,597	10/10	57,597	0.02	57,597	5.92
E1k.9	1,000	68,420	10/10	68,420	0.02	68,420	6.58
E31k.0	31,623	12,419	10/10	12,419	78.39	12,419	1,192.12
E31k.1	31,623	15,169	10/10	15,169	18.43	15,169	1,194.92
E3k.0	3,162	39,854	10/10	39,854	0.16	39,854	17.68
E3k.1	3,162	37,500	10/10	37,500	0.17	37,500	16.97
E3k.2	3,162	35,145	10/10	35,145	0.17	35,145	16.57
E3k.3	3,162	44,428	10/10	44,428	0.16	44,428	18.42
E3k.4	3,162	36,621	10/10	36,621	0.74	36,621	16.96
M10k.0	10,000	1,189	10/10	1,189	19.26	1,189	255.15
M1k.0	1,000	9,328	10/10	9,328	0.18	9,328	17.77
M1k.1	1,000	8,856	10/10	8,856	0.19	8,856	17.42
M1k.2	1,000	11,282	10/10	11,282	0.19	11,282	20.11
M1k.3	1,000	11,617	10/10	11,617	0.18	11,617	19.91
M3k.0	3,162	3,289	10/10	3,289	1.90	3,289	47.56
M3k.1	3,162	3,034	10/10	3,034	1.90	3,034	46.57

Table 4 *Results for symmetric JM random instances [13].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
ar9152	9,152	4,871	10/10	4,871	1.77	4,871	102.02
ca4663	4,663	13,768	10/10	13,768	1.21	13,768	35.18
dj89	89	437	10/10	437	0.00	437	0.29
eg7146	7,146	2,150	10/10	2,150	2.27	2,150	65.86
ei8246	8,246	124	10/10	124	1.44	124	65.69
fi10639	10,639	768	10/10	768	1.73	768	106.56
gr9882	9,882	1,399	10/10	1,399	1.99	1,399	97.15
ho14473	14,473	440	10/10	440	7.66	440	208.38
it16862	16,862	1,499	10/10	1,499	10.71	1,499	431.88
ja9847	9,847	6,413	10/10	6,413	4.71	6,413	101.17
kz9976	9,976	1,602	10/10	1,602	1.89	1,602	103.94
lu980	980	44	10/10	44	0.02	44	5.97
mo14185	14,185	939	10/10	939	10.88	939	180.20
mu1979	1,979	1,153	10/10	1,153	0.09	1,153	14.10
nu3496	3,496	650	10/10	650	0.35	650	24.84
pm8079	8,079	331	10/10	331	2.39	331	75.56
qa194	194	370	10/10	370	0.00	370	1.13
rw1621	1,621	150	10/10	150	0.09	150	11.46
sw24978	24,978	1,068	10/10	1,068	37.19	1,068	8,270.68
tz6117	6,117	486	10/10	486	0.74	486	40.26
uy734	734	389	10/10	389	0.04	389	5.04
vm22775	22,775	388	10/10	388	9.94	388	477.76
wi29	29	2,250	10/10	2,250	0.00	2,250	0.02
ym7663	7,663	3,113	10/10	3,113	2.74	3,113	66.67
zi929	929	887	10/10	887	0.02	887	8.12

Table 5 *Results for National TSP instances [14].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
bbz25234	25,234	15	10/10	15	12.10	15	467.37
bch2762	2,762	15	10/10	15	0.15	15	11.83
bck2217	2,217	16	10/10	16	0.19	16	10.27
bcl380	380	16	10/10	16	0.00	16	2.15
beg3293	3,293	24	10/10	24	0.20	24	15.00
bgb4355	4,355	13	10/10	13	0.35	13	23.01
bgd4396	4,396	28	10/10	28	0.35	28	21.27
bgf4475	4,475	25	10/10	25	0.36	25	21.28
bnd7168	7,168	15	10/10	15	2.49	15	41.90
boa28924	28,924	18	10/10	18	16.29	18	641.48
bva2144	2,144	15	10/10	15	0.09	15	9.93
dbj2924	2,924	16	10/10	16	0.16	16	12.99
dca1389	1,389	18	10/10	18	0.07	18	8.39
dcb2086	2,086	21	10/10	21	0.18	21	10.00
dcc1911	1,911	16	10/10	16	0.07	16	9.45
dea2382	2,382	25	10/10	25	0.10	25	11.75
dga9698	9,698	15	10/10	15	1.74	15	70.74
dhb3386	3,386	17	10/10	17	0.51	17	14.87
dja1436	1,436	15	10/10	15	0.04	15	7.72
djb2036	2,036	15	10/10	15	0.08	15	9.54
djc1785	1,785	24	10/10	24	0.06	24	9.63
dka1376	1,376	15	10/10	15	0.04	15	8.16
dkc3938	3,938	16	10/10	16	0.30	16	17.53
dkd1973	1,973	14	10/10	14	0.08	14	9.61
dke3097	3,097	19	10/10	19	0.37	19	13.75
dkf3954	3,954	20	10/10	20	0.72	20	17.57
dkg813	813	18	10/10	18	0.01	18	5.46
dlb3694	3,694	19	10/10	19	1.18	19	55.45
fdp3256	3,256	20	10/10	20	0.45	20	16.46
fea5557	5,557	15	10/10	15	1.89	15	36.32
fjr3672	3,672	21	10/10	21	0.23	21	17.65
fjs3649	3,649	21	10/10	21	0.22	21	17.01
fma21553	21,553	16	10/10	16	22.41	16	354.83
fnb1615	1,615	45	10/10	45	0.10	45	11.46
fnc19402	19,402	18	10/10	18	17.58	18	274.10
fqm5087	5,087	16	10/10	16	0.51	16	25.19
fra1488	1,488	13	10/10	13	0.04	13	8.23
frh19289	19,289	28	10/10	28	5.91	28	271.25
frv4410	4,410	15	10/10	15	0.83	15	19.00
fyg28534	28,534	15	10/10	15	43.24	15	587.37

Table 6 *Results for VLSI instances [14] (Part I).*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
icw1483	1,483	21	10/10	21	0.08	21	9.07
icx28698	28,698	18	10/10	18	16.39	18	594.93
ida8197	8,197	14	10/10	14	1.24	14	55.20
ido21215	21,215	16	10/10	16	7.50	16	345.75
ird29514	29,514	16	10/10	16	17.70	16	629.35
irw2802	2,802	15	10/10	15	0.33	15	12.52
irx28268	28,268	15	10/10	15	36.75	15	575.07
lap7454	7,454	16	10/10	16	1.02	16	45.54
ley2323	2,323	27	10/10	27	0.19	27	11.47
lim963	963	13	10/10	13	0.04	13	6.98
lsb22777	22,777	23	10/10	23	26.73	23	374.12
lsm2854	2,854	19	10/10	19	0.15	19	12.85
lsn3119	3,119	16	10/10	16	0.18	16	13.89
lta3140	3,140	17	10/10	17	0.19	17	13.95
ltb3729	3,729	13	10/10	13	0.31	13	17.97
mlt2597	2,597	21	10/10	21	0.12	21	11.98
pbd984	984	13	10/10	13	0.02	13	7.17
pbk411	411	14	10/10	14	0.01	14	2.54
pbl395	395	13	10/10	13	0.01	13	2.31
pbm436	436	15	10/10	15	0.01	15	2.70
pbn423	423	14	10/10	14	0.01	14	2.46
pds2566	2,566	15	10/10	15	0.26	15	11.14
pia3056	3,056	15	10/10	15	0.18	15	13.57
pjh17845	17,845	14	10/10	14	5.07	14	231.96
pka379	379	11	10/10	11	0.00	11	1.75
pma343	343	15	10/10	15	0.00	15	1.77
rbu737	737	13	10/10	13	0.02	13	4.62
rbv1583	1,583	18	10/10	18	0.11	18	8.68
rbw2481	2,481	27	10/10	27	0.23	27	11.96
rbx711	711	15	10/10	15	0.01	15	4.35
rby1599	1,599	18	10/10	18	0.12	18	9.58
xia16928	16,928	15	10/10	15	4.59	15	310.45
xit1083	1,083	11	10/10	11	0.02	11	6.46
xmc10150	10,150	17	10/10	17	1.56	17	239.83
xpr2308	2,308	14	10/10	14	0.10	14	10.09
xqc2175	2,175	15	10/10	15	0.20	15	10.18

Table 7 *Results for VLSI instances [14] (Part II).*

Beta	Gamma	*r*	Seed	Success	Lower bound	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
500	5,000	125	6,125	10/10	29	525	4.33	525	6,190.58
500	5,000	250	6,250	10/10	17	17	0.03	17	5.94
500	5,000	375	6,375	10/10	35	524	6.43	524	5,491.03
500	7,500	125	8,625	10/10	31	538	5.82	538	5,709.70
500	7,500	250	8,750	10/10	22	22	0.03	22	5.77
500	7,500	375	8,875	10/10	36	537	10.47	537	5,272.00
500	10,000	125	11,125	10/10	53	547	13.68	547	4,749.93
500	10,000	250	11,250	10/10	20	20	0.03	20	5.98
500	10,000	375	11,375	10/10	30	551	7.24	551	5,471.06
1,000	5,000	125	6,625	10/10	56	1,022	4.66	1,022	6,573.59
1,000	5,000	250	6,750	10/10	26	26	0.03	26	5.60
1,000	5,000	375	6,875	10/10	73	1,019	8.41	1,019	6,061.48
1,000	7,500	125	9,125	10/10	59	1,031	5.66	1,031	6,697.39
1,000	7,500	250	9,250	10/10	40	40	0.03	40	5.99
1,000	7,500	375	9,375	10/10	55	1,034	5.41	1,034	6,396.32
1,000	10,000	125	11,625	10/10	70	1,052	12.55	1,052	5,721.51
1,000	10,000	250	11,750	10/10	34	34	0.03	34	5.98
1,000	10,000	375	11,875	10/10	99	1,047	13.50	1,047	5,336.26
2,500	5,000	125	8,125	10/10	175	2,514	6.95	2,514	6,897.77
2,500	5,000	250	8,250	10/10	88	88	0.04	88	6.18
2,500	5,000	375	8,375	10/10	156	2,513	6.21	2,513	7,088.92
2,500	7,500	125	10,625	10/10	174	2,530	7.71	2,530	6,446.26
2,500	7,500	250	10,750	10/10	106	106	0.04	106	5.79
2,500	7,500	375	10,875	10/10	170	2,528	7.38	2,528	6,493.24
2,500	10,000	125	13,125	10/10	171	2,534	9.43	2,534	6,874.99
2,500	10,000	250	13,250	10/10	83	83	0.04	83	6.11
2,500	10,000	375	13,375	10/10	161	2,536	6.32	2,536	6,539.55

Table 8 *Results for hard instances with n = 500 [9].*

Beta	Gamma	r	Seed	Success	Lower bound	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
500	5,000	625	8,625	10/10	7	505	189.12	505	24,305.76
500	5,000	1,250	9,250	10/10	4	4	0.81	4	32.01
500	5,000	1,875	9,875	10/10	9	505	306.22	505	21,989.61
500	7,500	625	11,125	10/10	9	508	239.56	508	23,334.92
500	7,500	1,250	11,750	10/10	4	4	0.81	4	32.55
500	7,500	1,875	12,375	10/10	8	507	192.47	507	22,762.23
500	10,000	625	13,625	10/10	6	510	179.83	510	23,854.55
500	10,000	1250	14,250	10/10	4	4	0.81	4	32.44
500	10,000	1875	14,875	10/10	7	510	176.55	510	23,708.87
1,000	5,000	625	9,125	10/10	16	1,005	171.32	1,005	25,242.82
1,000	5,000	1,250	9,750	10/10	6	6	0.82	6	35.85
1,000	5,000	1,875	10,375	10/10	16	1,005	200.40	1,005	25,255.14
1,000	7,500	625	11,625	10/10	14	1,007	191.53	1,007	25,724.98
1,000	7,500	1,250	12,250	10/10	8	8	0.85	8	34.84
1,000	7,500	1,875	12,875	10/10	16	1,007	268.91	1,007	24,531.80
1,000	10,000	625	14,125	10/10	17	1,009	256.42	1,009	23,909.13
1,000	10,000	1,250	14,750	10/10	6	6	0.84	6	35.28
1,000	10,000	1,875	15,375	10/10	17	1,009	253.63	1,009	24,152.90
2,500	5,000	625	10,625	10/10	39	2,503	205.60	2,503	27,607.44
2,500	5,000	1,250	11,250	10/10	21	21	0.89	21	33.17
2,500	5,000	1,875	11,875	10/10	40	2,503	174.84	2,503	26,995.48
2,500	7,500	625	13,125	10/10	46	2,506	256.41	2,506	25,895.66
2,500	7,500	1,250	13,750	10/10	22	22	0.89	22	36.52
2,500	7,500	1,875	14,375	10/10	35	2,506	196.41	2,506	27,706.35
2,500	10,000	625	15,625	10/10	38	2,509	264.72	2,509	26,657.82
2,500	10,000	1,250	16,250	10/10	17	17	1.01	17	37.25
2,500	10,000	1,875	16,875	10/10	37	2,508	273.14	2,508	26,699.86

Table 9 *Results for hard instances with n = 2,500 [9].*

Name	Size	Lower bound	Lower bound time	BLKH Best value	BLKH Total time
pla33810	33,810	30,266	96.64	30,266	119.46
pla85900	85,900	51,006	723.77	51,006	953.31
ch71009	71,009	1,509	345.69	1,509	454.82
usa115475	115,475	1,110	115.80	1,110	404.64
C31k.1	31,623	65,079	46.16	65,079	68.61
C100k.0	100,000	44,745	560.56	44,745	848.44
C316k.0	316,228	23,159	8,855.63	23,159	11,137.78
E100k.0	100,000	7,169	73.84	7,169	280.10
E316k.0	316,228	4,382	960.17	4,382	3,203.02
E1M.0	1,000,000	2,396	10,605.88	2,396	33,543.05
ara238025	238,025	161	6,004.04	161	7,304.01
bby34656	34,656	18	8.67	18	26.02
bna56769	56,769	16	145.53	16	201.92
dan59296	59,296	23	30.67	23	93.19
fht47608	47,608	16	93.52	16	130.93
fna52057	52,057	16	119.42	16	165.59
fry33203	33,203	20	45.03	20	60.43
ics39603	39,603	18	62.29	18	86.10
lra498378	498,378	1,341	27,596.81	1,341	34,519.57
lrb744710	744,710	86	54,586.40	86	67,586.45
pbh30440	30,440	26	6.15	26	18.52
rbz43748	43,748	14	82.55	14	112.90
sra104815	104,815	194	662.00	194	909.87
xib32892	32,892	19	41.43	19	56.39
mona-lisa100K	100,000	340	36.74	340	241.80
vangogh120K	120,000	218	78.84	218	379.79
venus140K	140,000	185	120.95	185	537.75
pareja160K	160,000	283	133.94	283	687.35
courbet180K	180,000	230	194.70	230	900.96
earring200K	200,000	394	214.12	394	1,096.22

Table 10 *Results for very large symmetric instances [8][13][14].*

4.2 Performance on Asymmetric Instances

For all asymmetric instances, the following parameter settings for BLKH were chosen:

PROBLEM_FILE = ATSP_INSTANCES/<*instance name*>.atsp
INITIAL_PERIOD = 1000
MAX_CANDIDATES = 30
MAX_TRIALS = 1000
MOVE_TYPE = 3
OPTIMUM = <*best known cost*>
RUNS = 1

An explanation is given below:

PROBLEM_FILE: The symmetric test instances have been placed in the directory ATSP_INSTANCES and have filename extension ".atsp".

INITIAL_PERIOD: This parameter specifies the length of the first period in the Held-Karp ascent [16] (default is $n/2$).

MAX_CANDIDATES: This parameter is used to specify the size of the candidate sets used during the Lin-Kernighan search. Its value specifies the maximum number of candidate edges emanating from each vertex. The default value in LKH is 5. After some preliminary experiments, the value 30 was chosen.

MAX_TRIALS: Maximum number of trials (iterations) to be performed by the iterated Lin-Kernighan procedure (default is n).

MOVE_TYPE: Basic k-opt move type used in the Lin-Kernighan search (default is 5). As for symmetric instances, preliminary tests showed that 3-opt moves were sufficient for solving the asymmetric benchmark instances.

OPTIMUM: This parameter may be used to supply a best known solution cost. The algorithm will stop if this value is reached during the search process.

RUNS: Number of runs to be performed by LKH. Set to 1, since preliminary tests showed that optimum could be found for all benchmark instances using only one run (default is 10).

Tables 11-24 show the test results for the same asymmetric benchmark instances as were used by John LaRusic et al. in [10]. Each test was repeated ten times. The table column headers are the same as for the symmetric instances. As can be seen from the tables, BLKH was able to find new best values for 51 of the 332 instances. New best values are emphasized. Among the new best values, 44 are optimal (as they are equal to the calculated lower bounds). Notice also that BLKH runs considerably faster than JLR.

Table 25 shows computational results for some asymmetric real world instances not used in [10]. As can bee seen, BLKH found optimum solutions for all these instances.

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
amat100.0	100	50,981	10/10	50,981	0.01	50,981	0.13
amat100.1	100	26,551	10/10	26,551	0.01	50,821	0.16
amat100.2	100	50,821	10/10	50,821	0.01	63,674	0.13
amat100.3	100	21,695	10/10	21,695	0.01	62,994	0.17
amat100.4	100	63,674	10/10	63,674	0.02	55,534	0.15
amat100.5	100	17,539	10/10	17,539	0.01	44,548	0.15
amat100.6	100	62,994	10/10	62,994	0.01	72,650	0.15
amat100.7	100	19,952	10/10	19,952	0.01	59,291	0.15
amat100.8	100	55,534	10/10	55,534	0.01	55,462	0.13
amat100.9	100	20,779	10/10	20,779	0.01	49,928	0.13
amat316.10	316	17,207	10/10	17,207	0.04	21,896	0.98
amat316.11	316	59,291	10/10	59,291	0.10	20,451	1.25
amat316.12	316	43,608	10/10	43,608	0.04	26,551	0.88
amat316.13	316	55,462	10/10	55,462	0.07	21,695	1.04
amat316.14	316	9,978	10/10	9,978	0.04	17,539	2.64
amat316.15	316	49,928	10/10	49,928	0.04	19,952	2.07
amat316.16	316	10,624	10/10	10,624	0.04	20,779	1.18
amat316.17	316	21,896	10/10	21,896	0.04	26,165	0.95
amat316.18	316	6,715	10/10	6,715	0.04	17,207	1.71
amat316.19	316	20,451	10/10	20,451	0.04	43,608	2.36
amat1000.20	1,000	44,548	10/10	44,548	0.39	9,978	15.35
amat1000.21	1,000	26,165	10/10	26,165	0.39	10,624	15.81
amat1000.22	1,000	72,650	10/10	72,650	0.40	6,715	29.15
amat3162.30	3,162	2,985	10/10	**2,985**	3.99	47,237	5,760.72

Table 11 *Results for random asymmetric instances (amat) [13].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
coin100.0	100	253	10/10	253	0.01	253	14.49
coin100.1	100	219	10/10	219	0.01	219	18.85
coin100.2	100	203	10/10	203	0.23	203	16.18
coin100.3	100	232	10/10	232	0.01	232	20.84
coin100.4	100	214	10/10	214	0.01	214	0.16
coin100.5	100	244	10/10	244	0.01	244	18.60
coin100.6	100	239	10/10	239	0.00	239	9.68
coin100.7	100	265	10/10	265	0.00	265	20.47
coin100.8	100	198	10/10	198	0.12	198	15.54
coin100.9	100	243	10/10	243	0.01	243	21.38
coin316.10	316	227	10/10	227	0.04	227	82.78
coin316.11	316	238	10/10	238	0.04	238	87.09
coin316.12	316	225	10/10	225	0.05	225	72.60
coin316.13	316	245	10/10	245	0.14	245	66.07
coin316.14	316	278	10/10	278	0.03	278	88.84
coin316.15	316	282	10/10	282	0.04	282	56.75
coin316.16	316	243	10/10	243	0.06	243	47.92
coin316.17	316	277	10/10	277	0.03	277	88.05
coin316.18	316	277	10/10	277	0.04	277	66.54
coin316.19	316	259	10/10	259	0.03	259	0.53
coin1000.20	1,000	278	10/10	278	0.34	278	341.00
coin1000.21	1,000	327	10/10	327	0.32	327	422.52
coin1000.22	1,000	245	10/10	**245**	0.33	249	279.33
coin3162.30	3,162	260	10/10	**260**	3.85	649	4,075.57

Table 12 *Results for pay phone collection instances (coin) [13].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
crane100.0	100	173,390	10/10	173,390	0.01	173,390	0.12
crane100.1	100	152,923	10/10	152,923	0.14	180,146	30.65
crane100.2	100	214,843	10/10	214,843	0.01	214,843	0.14
crane100.3	100	145,622	10/10	145,622	0.01	145,622	0.28
crane100.4	100	171,484	10/10	171,484	0.01	171,484	0.26
crane100.5	100	205,739	10/10	205,739	0.01	205,739	0.12
crane100.6	100	185,071	10/10	185,071	0.01	185,071	0.17
crane100.7	100	200,173	10/10	200,173	0.01	200,173	0.12
crane100.8	100	205,258	10/10	205,258	0.01	205,258	0.18
crane100.9	100	192,987	10/10	192,987	0.01	192,987	23.40
crane316.10	316	119,345	10/10	119,345	0.99	120,333	136.68
crane316.11	316	108,045	10/10	108,045	0.04	108,045	1.08
crane316.12	316	132,750	10/10	132,750	0.04	132,750	2.34
crane316.13	316	102,898	10/10	102,898	0.05	102,898	0.95
crane316.14	316	145,963	10/10	145,963	0.04	145,963	0.73
crane316.15	316	128,548	10/10	128,548	0.05	128,548	192.00
crane316.16	316	111,939	10/10	111,939	0.04	111,939	1.71
crane316.17	316	102,571	10/10	102,571	0.04	102,571	1.42
crane316.18	316	106,821	10/10	106,821	0.05	106,821	1.92
crane316.19	316	133,399	10/10	133,399	0.04	133,399	0.86
crane1000.20	1,000	56,343	10/10	56,343	0.42	56,343	19.44
crane1000.21	1,000	61,720	10/10	64,450	28.83	64,450	722.66
crane1000.22	1,000	58,091	10/10	58,466	4.01	58,466	489.04
crane3162.30	3,162	41,751	10/10	**41,751**	4.31	76,894	6,234.88

Table 13 *Results for random Euclidean stacker crane instances (crane) [13].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
disk100.0	100	508,034	10/10	508,034	0.01	508,034	0.12
disk100.1	100	473,495	10/10	473,495	0.01	473,495	0.10
disk100.2	100	382,677	10/10	382,677	0.11	386,809	10.39
disk100.3	100	453,657	10/10	453,657	0.01	453,657	0.15
disk100.4	100	415,696	10/10	415,696	0.01	415,696	0.19
disk100.5	100	553,069	10/10	553,069	0.01	553,069	0.21
disk100.6	100	432,563	10/10	432,563	0.01	432,563	0.13
disk100.7	100	550,543	10/10	550,543	0.01	550,543	0.16
disk100.8	100	396,159	10/10	396,159	0.01	396,159	0.40
disk100.9	100	426,697	10/10	426,697	0.01	426,697	0.22
disk316.10	316	309,801	10/10	309,801	0.05	309,801	0.82
disk316.11	316	259,308	10/10	259,308	0.08	259,308	1.33
disk316.12	316	273,516	10/10	273,516	0.05	273,516	1.41
disk316.13	316	236,394	10/10	236,394	0.18	236,394	32.21
disk316.14	316	226,920	10/10	226,920	0.05	226,920	2.64
disk316.15	316	271,724	10/10	271,724	0.05	271,724	1.50
disk316.16	316	249,590	10/10	249,590	0.15	249,590	7.44
disk316.17	316	305,813	10/10	305,813	0.05	305,813	1.01
disk316.18	316	246,356	10/10	246,356	0.19	246,356	1.37
disk316.19	316	320,680	10/10	320,680	0.18	320,680	1.16
disk1000.20	1,000	190,741	10/10	190,741	0.46	190,741	975.74
disk1000.21	1,000	190,665	10/10	190,665	0.45	190,665	746.68
disk1000.22	1,000	171,509	10/10	**171,509**	3.79	195,825	1,537.19
disk3162.30	3,162	114,028	10/10	**114,028**	8.52	275,891	5,957.56

Table 14 *Results for disk drive instances (disk) [13].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
rect100.0	100	223,717	10/10	223,717	0.01	223,717	12.40
rect100.1	100	296,349	10/10	296,349	0.01	296,349	28.72
rect100.2	100	198,319	10/10	198,319	0.01	198,319	22.94
rect100.3	100	252,297	10/10	252,297	0.01	252,297	24.45
rect100.4	100	246,494	10/10	246,494	0.01	246,494	27.80
rect100.5	100	270,406	10/10	270,406	0.01	270,406	22.01
rect100.6	100	226,069	10/10	226,069	0.01	226,069	18.40
rect100.7	100	245,457	10/10	245,457	0.01	245,457	21.60
rect100.8	100	259,347	10/10	259,347	0.01	259,347	27.80
rect100.9	100	199,114	10/10	199,114	0.24	200,000	22.88
rect316.10	316	160,358	10/10	160,358	0.04	160,358	168.87
rect316.11	316	133,083	10/10	133,083	0.07	133,083	121.94
rect316.12	316	143,209	10/10	143,209	0.05	143,209	134.99
rect316.13	316	116,995	10/10	116,995	0.05	116,995	104.04
rect316.14	316	140,437	10/10	140,437	0.05	140,437	158.32
rect316.15	316	144,858	10/10	144,858	0.05	144,858	118.90
rect316.16	316	142,029	10/10	142,029	0.05	142,029	110.67
rect316.17	316	121,054	10/10	121,054	0.05	121,054	110.10
rect316.18	316	166,616	10/10	166,616	0.05	166,616	217.73
rect316.19	316	142,134	10/10	142,134	0.05	142,134	151.49
rect1000.20	1,000	88,925	10/10	88,925	0.44	88,925	521.58
rect1000.21	1,000	82,241	10/10	82,241	0.45	82,241	543.19
rect1000.22	1,000	73,643	10/10	73,643	0.47	73,643	398.07
rect3162.30	3,162	47,063	10/10	**47,063**	5.21	123,706	5,241.82

Table 15 *Results for random two-dimensional rectilinear instances (rect) [13].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
rtilt100.0	100	260,342	10/10	**260,342**	0.01	286,962	38.31
rtilt100.1	100	291,040	10/10	291,040	0.01	291,040	33.51
rtilt100.2	100	227,248	10/10	**227,248**	0.01	270,913	43.60
rtilt100.3	100	236,920	10/10	**236,920**	0.01	288,191	47.37
rtilt100.4	100	294,367	10/10	**294,367**	0.01	307,304	45.41
rtilt100.5	100	238,332	10/10	**238,332**	0.01	280,052	42.26
rtilt100.6	100	276,224	10/10	**276,224**	0.01	276,947	33.44
rtilt100.7	100	361,152	10/10	361,152	0.01	361,152	0.33
rtilt100.8	100	368,500	10/10	368,500	0.01	368,500	1.24
rtilt100.9	100	198,376	10/10	**198,376**	1.55	307,809	50.76
rtilt316.10	316	152,510	10/10	**152,510**	0.05	261,362	431.86
rtilt316.11	316	139,280	10/10	**139,280**	0.07	240,201	392.59
rtilt316.12	316	162,936	10/10	**162,936**	0.07	272,157	337.92
rtilt316.13	316	141,912	10/10	**141,912**	0.06	249,325	335.80
rtilt316.14	316	157,594	10/10	**157,594**	0.06	281,467	361.11
rtilt316.15	316	181,676	10/10	**181,676**	0.05	231,232	395.26
rtilt316.16	316	173,991	10/10	**173,991**	0.07	273,055	358.35
rtilt316.17	316	128,217	10/10	**128,217**	1.23	317,338	276.98
rtilt316.18	316	147,764	10/10	**147,764**	0.06	263,831	334.85
rtilt316.19	316	133,664	10/10	**133,664**	0.11	270,363	387.19
rtilt1000.20	1,000	86,549	10/10	**86,549**	0.68	287,949	1,130.05
rtilt1000.21	1,000	86,828	10/10	**86,828**	0.70	323,513	1,329.22
rtilt1000.22	1,000	92,692	10/10	**92,692**	0.60	319,739	1,218.79
rtilt3162.30	3,162	47,152	10/10	**47,152**	9.35	391,531	3,980.61

Table 16 *Results for tilted drilling machine instances, additive norm (rtilt) [13].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
shop100.0	100	2,232	10/10	2,232	0.01	2,232	8.89
shop100.1	100	2,608	10/10	2,608	0.01	2,608	0.16
shop100.2	100	3,620	10/10	3,620	0.01	3,620	0.10
shop100.3	100	2,526	10/10	2,526	0.01	2,526	0.40
shop100.4	100	2,792	10/10	2,792	0.01	2,792	0.10
shop100.5	100	2,679	10/10	2,679	0.01	2,679	0.09
shop100.6	100	2,314	10/10	2,314	0.01	2,314	1.01
shop100.7	100	2,665	10/10	2,665	0.01	2,665	0.09
shop100.8	100	2,621	10/10	2,621	0.01	2,621	0.11
shop100.9	100	2,276	10/10	2,276	0.01	2,276	7.46
shop316.10	316	2,311	10/10	2,311	0.04	2,311	119.55
shop316.11	316	2,907	10/10	2,907	0.05	2,907	0.42
shop316.12	316	2,541	10/10	2,541	0.10	2,541	0.40
shop316.13	316	2,970	10/10	2,970	0.05	2,970	0.42
shop316.14	316	2,235	10/10	2,235	0.09	2,235	4.38
shop316.15	316	2,459	10/10	2,459	0.05	2,459	0.70
shop316.16	316	2,806	10/10	2,806	0.04	2,806	0.43
shop316.17	316	2,298	10/10	2,298	0.04	2,298	5.34
shop316.18	316	2,204	10/10	2,204	0.04	2,204	4.23
shop316.19	316	2,811	10/10	2,811	0.04	2,811	0.44
shop1000.20	1,000	2,041	10/10	**2,041**	0.48	2,190	719.28
shop1000.21	1,000	2,709	10/10	2,709	7.28	2,709	1.83
shop1000.22	1,000	2,057	10/10	**2,057**	2.16	2,184	730.85
shop3162.30	3,162	2,407	10/10	2,407	5.49	2,407	127.45

Table 17 *Results for no-wait flowshop instances (shop) [13].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
smat100.0	100	70,175	10/10	70,175	0.01	70,175	24.42
smat100.1	100	69,636	10/10	69,636	0.01	69,636	20.17
smat100.2	100	59,186	10/10	59,186	0.01	59,186	28.64
smat100.3	100	73,104	10/10	73,104	0.01	73,104	24.16
smat100.4	100	58,962	10/10	58,962	0.01	58,962	18.51
smat100.5	100	89,297	10/10	89,297	0.01	89,297	30.22
smat100.6	100	57,248	10/10	57,248	0.11	61,904	18.74
smat100.7	100	67,624	10/10	67,624	0.01	67,624	28.50
smat100.8	100	66,698	10/10	66,698	0.01	66,698	22.76
smat100.9	100	71,811	10/10	71,811	0.01	71,811	25.26
smat316.10	316	21,672	10/10	21,672	0.05	21,672	147.15
smat316.11	316	23,998	10/10	23,998	0.05	23,998	147.35
smat316.12	316	34,266	10/10	34,266	0.04	34,266	152.54
smat316.13	316	27,251	10/10	27,251	0.04	27,251	130.37
smat316.14	316	23,203	10/10	23,203	0.04	23,203	119.50
smat316.15	316	24,252	10/10	24,252	0.04	24,252	152.07
smat316.16	316	28,690	10/10	28,690	0.04	28,690	180.38
smat316.17	316	27,313	10/10	27,313	0.04	27,313	157.42
smat316.18	316	29,449	10/10	29,449	0.04	29,449	144.71
smat316.19	316	25,414	10/10	25,414	0.05	25,414	117.88
smat1000.20	1,000	10,371	10/10	10,371	0.43	10,371	826.19
smat1000.21	1,000	9,360	10/10	9,360	0.42	9,360	636.71
smat1000.22	1,000	8,817	10/10	8,817	0.43	8,817	572.56
smat3162.30	3,162	2,959	10/10	**2,959**	4.56	49,035	6,963.29

Table 18 *Results for random symmetric matrix instances (smat) [13].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
stilt100.0	100	382,208	10/10	**382,208**	0.01	404,808	31.09
stilt100.1	100	491,416	10/10	491,416	0.01	491,416	3.34
stilt100.2	100	377,720	10/10	**377,720**	0.01	392,728	35.26
stilt100.3	100	401,976	10/10	**401,976**	0.03	410,408	35.13
stilt100.4	100	347,440	10/10	**347,440**	0.02	389,296	45.20
stilt100.5	100	456,788	10/10	456,788	0.01	456,788	1.64
stilt100.6	100	417,056	10/10	417,056	0.01	417,056	1.10
stilt100.7	100	494,360	10/10	494,360	0.01	494,360	2.12
stilt100.8	100	522,748	10/10	522,748	0.01	522,748	0.49
stilt100.9	100	321,884	10/10	**334,622**	0.21	383,838	38.75
stilt316.10	316	226,504	10/10	**226,504**	0.21	401,968	336.37
stilt316.11	316	235,910	10/10	**235,910**	0.07	424,144	268.78
stilt316.12	316	291,320	10/10	**291,320**	0.05	432,260	386.30
stilt316.13	316	179,462	10/10	**186,488**	2.78	365,846	302.48
stilt316.14	316	198,152	10/10	**205,084**	7.80	430,308	281.13
stilt316.15	316	232,104	10/10	**235,092**	1.10	379,108	298.41
stilt316.16	316	290,738	10/10	**290,738**	0.05	370,896	372.58
stilt316.17	316	196,896	10/10	**204,572**	2.80	391,892	247.78
stilt316.18	316	244,688	10/10	**244,688**	0.05	418,088	355.59
stilt316.19	316	212,268	10/10	**219,880**	1.22	367,686	287.51
stilt1000.20	1,000	121,812	10/10	**121,812**	0.56	466,364	986.35
stilt1000.21	1,000	117,542	10/10	**118,344**	19.97	460,732	1,295.38
stilt1000.22	1,000	127,000	10/10	**127,000**	1.39	456,270	1,236.09
stilt3162.30	3,162	66,552	10/10	**66,552**	7.42	569,890	4,127.88

Table 19 *Results for tilted drilling machine instances, sup norm (stilt) [13].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
super100.0	100	10	10/10	10	0.00	10	0.09
super100.1	100	11	10/10	11	0.00	11	0.09
super100.2	100	10	10/10	10	0.00	10	0.09
super100.3	100	10	10/10	10	0.00	10	0.09
super100.4	100	10	10/10	10	0.00	10	0.09
super100.5	100	10	10/10	10	0.00	10	0.09
super100.6	100	10	10/10	10	0.00	10	0.09
super100.7	100	10	10/10	10	0.00	10	0.09
super100.8	100	10	10/10	10	0.00	10	0.09
super100.9	100	10	10/10	10	0.00	10	0.09
super316.10	316	9	10/10	9	0.02	9	0.49
super316.11	316	9	10/10	9	0.02	9	0.47
super316.12	316	9	10/10	9	0.02	9	0.48
super316.13	316	9	10/10	9	0.02	9	0.48
super316.14	316	9	10/10	9	0.02	9	0.48
super316.15	316	9	10/10	9	0.04	9	0.52
super316.16	316	9	10/10	9	0.02	9	0.48
super316.17	316	9	10/10	9	0.02	9	0.47
super316.18	316	9	10/10	9	0.02	9	0.49
super316.19	316	9	10/10	9	0.02	9	0.49
super1000.20	1,000	8	10/10	8	0.19	8	12.07
super1000.21	1,000	8	10/10	8	0.19	8	12.07
super1000.22	1,000	8	10/10	8	0.20	8	10.63
super3162.30	3,162	7	10/10	**7**	1.95	9	1,905.87

Table 20 *Results for approximate shortest common superstring instances (super) [13].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
tmat100.0	100	50,981	10/10	50,981	0.01	50,981	0.11
tmat100.1	100	50,821	10/10	50,821	0.01	50,821	0.12
tmat100.2	100	63,674	10/10	63,674	0.01	63,674	0.11
tmat100.3	100	62,994	10/10	62,994	0.01	62,994	0.13
tmat100.4	100	55,534	10/10	55,534	0.01	55,534	0.15
tmat100.5	100	35,793	10/10	35,793	0.01	35,793	0.12
tmat100.6	100	72,650	10/10	72,650	0.01	72,650	0.22
tmat100.7	100	59,291	10/10	59,291	0.01	59,291	0.11
tmat100.8	100	55,462	10/10	55,462	0.01	55,462	0.14
tmat100.9	100	49,928	10/10	49,928	0.01	49,928	0.17
tmat316.10	316	21,896	10/10	21,896	0.06	21,896	0.58
tmat316.11	316	18,240	10/10	18,240	0.04	18,240	0.59
tmat316.12	316	26,551	10/10	26,551	0.07	26,551	0.48
tmat316.13	316	20,090	10/10	20,090	0.05	20,090	0.62
tmat316.14	316	17,539	10/10	17,539	0.04	17,539	0.70
tmat316.15	316	19,952	10/10	19,952	0.05	19,952	0.59
tmat316.16	316	20,779	10/10	20,779	0.05	20,779	0.61
tmat316.17	316	26,165	10/10	26,165	0.07	26,165	0.56
tmat316.18	316	17,207	10/10	17,207	0.04	17,207	0.62
tmat316.19	316	43,608	10/10	43,608	0.08	43,608	0.48
tmat1000.20	1,000	9,978	10/10	9,978	1.34	9,978	2.02
tmat1000.21	1,000	10,624	10/10	10,624	1.48	10,624	1.84
tmat1000.22	1,000	6,715	10/10	6,715	0.44	6,715	2.11
tmat3162.30	3,162	2,985	10/10	2,985	5.91	2,985	12.95

Table 21 *Results for shortest-path closure of amat (tmat) [13].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
tsmat100.0	100	44,258	10/10	44,258	0.01	44,258	44.31
tsmat100.1	100	42,415	10/10	42,415	0.01	42,415	43.65
tsmat100.2	100	37,786	10/10	37,786	0.01	37,786	34.48
tsmat100.3	100	40,608	10/10	40,608	0.01	40,608	36.24
tsmat100.4	100	48,184	10/10	48,184	0.01	48,184	40.81
tsmat100.5	100	54,108	10/10	54,108	0.01	54,108	42.30
tsmat100.6	100	54,157	10/10	54,486	6.16	54,486	46.66
tsmat100.7	100	45,189	10/10	45,189	0.01	45,189	41.46
tsmat100.8	100	64,065	10/10	64,065	0.01	64,065	29.83
tsmat100.9	100	61,244	10/10	61,244	0.01	61,244	43.66
tsmat316.10	316	20,537	10/10	20,537	0.06	20,537	239.36
tsmat316.11	316	22,643	10/10	22,643	0.06	22,643	287.11
tsmat316.12	316	21,337	10/10	21,337	0.06	21,337	276.18
tsmat316.13	316	24,463	10/10	24,463	0.06	24,463	206.42
tsmat316.14	316	21,042	10/10	21,042	0.06	21,042	240.87
tsmat316.15	316	21,767	10/10	21,767	0.05	21,767	208.61
tsmat316.16	316	28,690	10/10	28,690	0.07	28,690	225.90
tsmat316.17	316	27,099	10/10	27,099	0.07	27,099	247.76
tsmat316.18	316	24,829	10/10	24,829	0.07	24,829	178.47
tsmat316.19	316	15,023	10/10	15,023	0.04	15,023	263.81
tsmat1000.20	1,000	8104	10/10	8,104	1.03	8,104	291.85
tsmat1000.21	1,000	6823	10/10	6,823	0.51	6,823	886.82
tsmat1000.22	1,000	7578	10/10	7,578	0.61	7,578	658.42
tsmat3162.30	3,162	2544	10/10	2,544	6.08	2,544	1,885.18

Table 22 *Results for shortest-path closure of smat (tsmat) [13].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
br17	17	5	10/10	5	0.00	5	0.02
ft53	53	379	10/10	379	0.00	379	0.06
ft70	70	976	10/10	976	0.01	976	9.81
ftv33	34	143	10/10	143	0.00	143	0.02
ftv35	36	154	10/10	154	0.00	154	0.02
ftv38	39	154	10/10	154	0.00	154	0.03
ftv44	45	162	10/10	162	0.00	162	0.03
ftv47	48	168	10/10	168	0.00	168	0.03
ftv55	56	154	10/10	154	0.00	154	0.05
ftv64	65	160	10/10	160	0.00	160	0.05
ftv70	71	161	10/10	161	0.01	161	0.09
ftv90	91	148	10/10	148	0.00	148	22.67
ftv100	101	155	10/10	155	0.00	155	23.17
ftv110	111	165	10/10	165	0.01	165	21.22
ftv120	121	165	10/10	165	0.01	165	0.87
ftv130	131	172	10/10	172	0.02	172	0.17
ftv140	141	172	10/10	172	0.01	172	0.46
ftv150	151	178	10/10	178	0.20	178	0.23
ftv160	161	178	10/10	178	0.26	178	0.19
ftv170	171	180	10/10	180	0.20	180	0.94
kro124p	100	2,347	10/10	2,347	0.01	2,347	0.15
p43	43	17	10/10	17	0.00	17	2.12
rbg323	323	23	10/10	23	0.09	23	78.19
rbg358	358	21	10/10	21	0.09	21	4.49
rbg403	403	19	10/10	19	0.21	19	43.47
rbg443	443	18	10/10	18	0.26	18	8.90
ry48p	48	1,232	10/10	1,232	0.84	1,232	11.43

Table 23 *Results for asymmetric TSPLIB instances [8].*

Name	Size	Lower bound	Success	BLKH Best value	BLKH Total time	JLR Best value	JLR Total time
balas84	84	18	10/10	18	0.00	18	0.07
balas108	108	13	10/10	13	0.01	13	0.09
balas120	120	22	10/10	22	0.01	22	0.11
balas160	160	13	10/10	13	0.01	13	0.17
balas200	200	13	10/10	13	0.02	13	0.39
ran500.0	500	24	10/10	24	0.17	24	1.23
ran500.1	500	22	10/10	22	0.17	22	3.73
ran500.2	500	23	10/10	23	0.17	23	2.73
ran500.3	500	23	10/10	23	0.17	23	2.43
ran500.4	500	28	10/10	28	0.17	28	1.89
ran1000.0	1,000	18	10/10	18	0.67	18	4.61
ran1000.1	1,000	17	10/10	17	0.67	17	7.52
ran1000.2	1,000	17	10/10	17	0.67	17	9.46
ran1000.3	1,000	19	10/10	19	0.67	19	8.25
ran1000.4	1,000	19	10/10	19	0.67	19	7.61
ftv180	181	35	10/10	37	0.25	37	25.08
uk66	66	170	10/10	170	0.00	170	0.05

Table 24 *Results for 5 asymmetric scheduling instances created by Egon Balas, and 10 asymmetric random and 2 real world instances created by Fischetti [15].*

Name	Size	Success	Lower bound	BLKH Best value	BLKH Total time
atex1	16	10/10	747	747	0.00
atex3	32	10/10	306	306	0.00
atex4	48	10/10	306	306	0.00
atex5	72	10/10	306	306	0.00
atex8	600	10/10	306	306	0.12
big702	702	10/10	358	358	4.91
code198	198	10/10	3,210	3,210	0.02
code253	253	10/10	12,220	12,220	0.04
dc112	112	10/10	155	155	0.01
dc126	126	10/10	3,084	3,084	0.01
dc134	134	10/10	69	69	0.01
dc176	176	10/10	159	159	0.02
dc188	188	10/10	192	192	0.02
dc563	563	10/10	143	143	0.32
dc849	849	10/10	63	63	1.03
dc895	895	10/10	833	833	1.22
dc932	932	10/10	2,699	2,699	1.34
td100.1	101	10/10	5,095	5,095	0.01
td316.10	317	10/10	5,036	5,036	0.04
td1000.20	1,001	10/10	4,824	4,824	0.47

Table 25 *Results for 20 asymmetric real world instances [13].*

5. The Maximum Scatter Traveling Salesman Problem

A problem closely related to the BTSP is the Maximum Scatter Traveling Salesman Problem (MSTSP). The MSTSP asks for a Hamiltonian cycle in which the smallest edge is as large possible. Applications of MSTSP include medical image processing [17].

Figure 2 shows an MSTSP tour for usa1097. Its total length is 2,251,150,121 meters, and its smallest edge has a length of 2,353,533 meters.

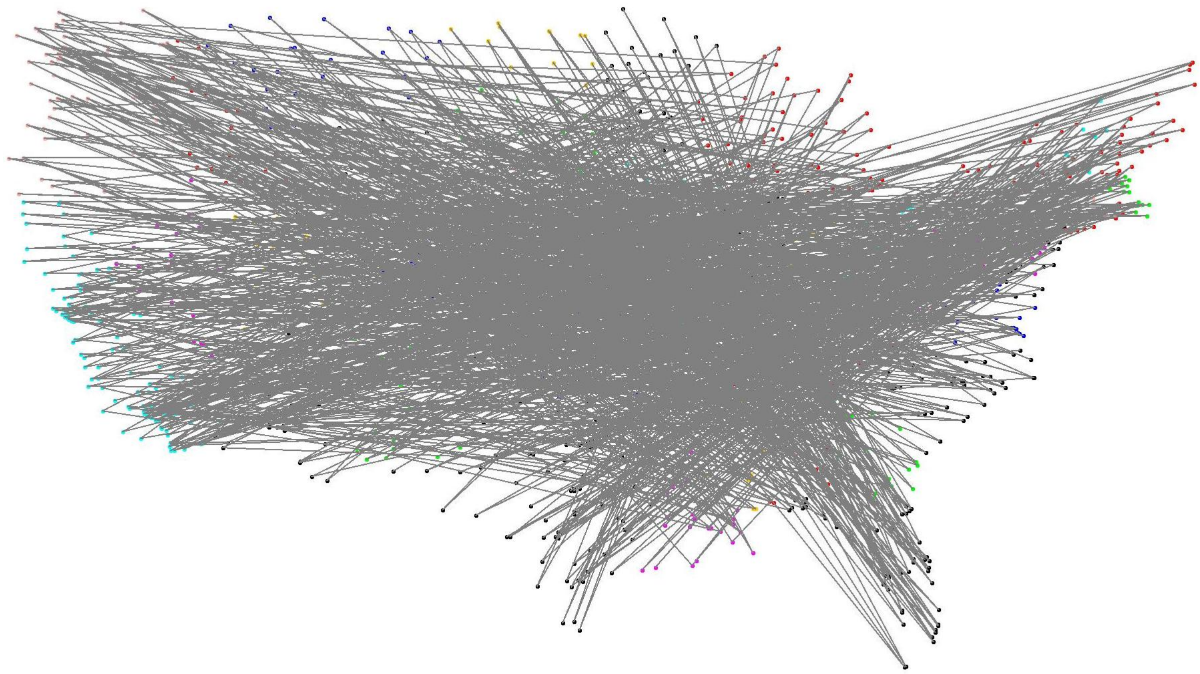

Figure 1 *MSTSP tour for usa1097.*

It is well known that the MSTSP on cost matrix c_{ij} can be formulated as a BTSP using the transformation $c'_{ij} = M - c_{ij}$, where M is $\max\{c_{ij}\}$. Thus, BLKH can be used for solving the MSTSP after a minor revision.

The performance of this revised version, called MBLKH, has been evaluated on the same asymmetric instances as used by John LaRusic et al. [10]. The computational results are compared in Tables 26-27. As can be seen, the same best values are found, but MBLKH uses much less CPU time.

The performance of MBLKH has also been evaluated on symmetric instances. The computational results for all symmetric TSPLIB instances with up to 18,512 vertices are shown in Tables 28-29. It is to be noted that whereas the asymmetric instances were quickly solved by MBLKH, solution of some of the symmetric instances consume considerably long CPU time, and the best value is found in only 1 out of 10 tests (Success = 1/10). Symmetric MSTSP instances are challenging for MBLKH and further research is needed if very large instances of this type are to be solved in a reasonable time.

Name	Size	Upper bound	Success	MBLKH Best value	MBLKH Total time	JLR Best value	JLR Total time
coin100.0	100	896	10/10	891	10.81	891	40.99
coin100.1	100	858	10/10	858	0.02	858	14.51
coin100.2	100	974	10/10	974	0.03	974	34.58
coin100.3	100	890	10/10	890	0.01	890	34.12
coin100.4	100	903	10/10	903	0.01	903	10.90
coin316.10	316	1684	10/10	1684	0.11	1684	293.33
crane100.0	100	647,354	10/10	647,354	0.01	647,354	0.30
crane100.1	100	627,751	10/10	627,751	0.01	627,751	0.11
crane100.2	100	645,372	10/10	645,372	0.01	645,372	0.22
crane100.3	100	629,932	10/10	629,932	0.01	629,932	36.57
crane100.4	100	619,054	10/10	619,054	0.01	619,054	0.23
crane316.10	316	664,713	10/10	664,713	0.10	664,713	385.49
disk100.0	100	4,767,083	10/10	4,767,083	0.98	4,767,083	78.04
disk100.1	100	4,882,684	10/10	4,882,684	0.99	4,882,684	31.31
disk100.2	100	4,959,789	10/10	4,959,789	0.01	4,959,789	1.24
disk100.3	100	4,663,663	10/10	4,663,663	0.08	4,663,663	57.06
disk100.4	100	49,971	10/10	4,849,971	0.02	4,849,971	57.94
disk316.10	316	4,947,068	10/10	4,947,068	0.27	4,947,068	416.43
rtilt100.0	100	529,202	10/10	529,202	0.01	529,202	0.24
rtilt100.1	100	546,234	10/10	546,234	0.01	546,234	0.13
rtilt100.2	100	494,714	10/10	494,714	0.01	494,714	0.16
rtilt100.3	100	500,897	10/10	500,897	0.01	500,897	0.13
rtilt100.4	100	517,383	10/10	517,383	0.01	517,383	0.12
rtilt316.1	316	509,367	10/10	509,367	0.13	509,367	1.06
shop100.0	100	1939	10/10	1939	0.01	1939	6.12
shop100.1	100	1786	10/10	1786	0.01	1786	0.10
shop100.2	100	2466	10/10	2466	0.02	2466	57.33
shop100.3	100	2044	10/10	2044	0.01	2044	0.37
shop100.4	100	2104	10/10	2104	0.01	2104	4.07
shop316.10	316	1966	10/10	1966	0.15	1966	1.06
stilt100.0	100	1,011,282	10/10	1,011,282	0.01	1,011,282	0.11
stilt100.1	100	1,071,396	10/10	1,071,396	0.01	1,071,396	0.14
stilt100.2	100	957,076	10/10	957,076	0.01	957,076	0.11
stilt100.3	100	997,468	10/10	997,468	0.01	997,468	0.12
stilt100.4	100	985,154	10/10	985,154	0.01	985,154	0.12
stilt316.10	316	990,472	10/10	990,472	0.12	990,472	0.85
super100.0	100	16	10/10	16	0.01	16	0.09
super100.1	100	16	10/10	16	0.01	16	0.09
super100.2	100	1	10/10	61	0.01	61	0.09
super100.3	100	16	10/10	16	0.01	16	0.09
super100.4	100	16	10/10	16	0.01	16	0.10
super316.10	316	17	10/10	17	0.04	17	0.49
ftv180	181	180	10/10	180	0.05	180	0.34
uk66	66	609	10/10	604	0.47	604	12.52

Table 26 *Results for asymmetric MSTSP instances [10] (Part I).*

Name	Size	Upper bound	Success	MBLKH Best value	MBLKH Total time	JLR Best value	JLR Total time
balas84	84	84	10/10	29	0.01	29	0.06
balas108	108	108	10/10	24	0.01	24	0.09
balas120	120	120	10/10	29	0.01	29	1.04
balas160	160	160	10/10	31	0.02	31	0.18
balas200	200	200	10/10	32	0.03	32	0.23
ran500.0	500	500	10/10	1,000	0.12	1,000	2.06
ran500.1	500	500	10/10	997	0.11	997	1.28
ran500.2	500	500	10/10	997	0.11	997	1.43
ran500.3	500	500	10/10	998	0.11	998	1.92
ran500.4	500	500	10/10	996	0.12	996	1.08
ran1000.0	1,000	1,000	10/10	1,000	0.45	1,000	9.79
ran1000.1	1,000	1,000	10/10	1,005	0.44	1,005	7.55
ran1000.2	1,000	1,000	10/10	1,004	0.44	1,004	5.82
ran1000.3	1,000	1,000	10/10	1,004	0.44	1,004	2.60
ran1000.4	1,000	1,000	10/10	1,006	0.44	1,006	50.92
br17	17	17	10/10	5	0.00	5	0.02
ft53	53	53	10/10	379	0.00	379	0.06
ft70	70	70	10/10	976	0.01	976	9.81
ftv33	34	34	10/10	143	0.00	143	0.02
ftv35	36	36	10/10	154	0.00	154	0.02
ftv38	39	39	10/10	154	0.00	154	0.03
ftv44	45	45	10/10	162	0.00	162	0.03
ftv47	48	48	10/10	168	0.00	168	0.03
ftv55	56	56	10/10	154	0.01	154	0.05
ftv64	65	65	10/10	160	0.01	160	0.05
ftv70	71	71	10/10	161	0.01	161	0.09
ftv90	91	91	10/10	148	0.01	148	22.67
ftv100	101	101	10/10	155	0.01	155	23.17
ftv110	111	111	10/10	165	0.01	165	21.22
ftv120	121	121	10/10	165	0.01	165	0.87
ftv130	131	131	10/10	172	0.02	172	0.17
ftv140	141	141	10/10	172	0.02	172	0.46
ftv150	151	151	10/10	178	0.02	178	0.23
ftv160	161	161	10/10	178	0.02	178	0.19
ftv170	171	171	10/10	180	0.03	180	0.94
kro124p	100	100	10/10	2,347	0.01	2,347	0.15
p43	43	43	10/10	17	0.00	17	2.12
rbg323	323	323	10/10	23	0.11	23	78.19
rbg358	358	358	10/10	21	0.15	21	4.49
rbg403	403	403	10/10	19	0.21	19	43.47
rbg443	443	443	10/10	18	0.27	18	8.90
ry48p	48	48	10/10	1,232	0.01	1,232	11.43

Table 27 *Results for asymmetric MSTSP instances [10] (Part II).*

Name	Size	Upper bound	Success	MBLKH Best value	MBLKH Total time
a280	280	148	10/10	**148**	0.03
ali535	535	14,464	10/10	5,741	156.87
att48	48	1,103	10/10	**1,103**	0.34
att532	532	1,423	10/10	897	183.57
bayg29	29	189	10/10	**189**	0.00
bays29	29	235	10/10	231	0.02
berlin52	52	859	10/10	541	0.57
bier127	127	9,656	10/10	4,828	4.83
brazil58	58	3,289	10/10	1,906	1.22
brd14051	14,051	4,470	1/10	3,808	32,101.38
brg180	180	9,000	10/10	**9,000**	0.02
burma14	14	514	10/10	498	0.00
ch130	130	458	10/10	**458**	0.01
ch150	150	454	10/10	**454**	0.01
d1291	1,291	1,916	2/10	1,785	182.68
d15112	15,112	12,527	2/10	11,783	20,281.49
d1655	1,655	1,741	10/10	**1,741**	0.73
d18512	18,512	4,453	4/10	4,269	17,146.56
d198	198	1,918	10/10	738	22.18
d2103	2,103	2,095	6/10	2,042	50.25
d493	493	1,449	10/10	824	108.93
d657	657	1,836	2/10	1,781	26.28
dantzig42	42	102	10/10	73	0.16
dsj1000	1,000	808,681	6/10	716,294	349.22
eil101	101	46	10/10	45	0.02
eil51	51	41	10/10	39	0.06
eil76	76	41	10/10	**41**	0.00
fl1400	1,400	1,299	10/10	1,261	14.32
fl1577	1,577	1,203	1/10	1,051	546.45
fl3795	3,795	1,287	1/10	1,175	1,173.81
fl417	417	1,262	10/10	**1,262**	0.06
fnl4461	4,461	2,641	1/10	2,537	716.99
fri26	26	129	10/10	102	0.02
gil262	262	131	10/10	129	0.14
gr120	120	563	10/10	**563**	0.01
gr137	137	7,618	10/10	5,398	3.13
gr17	17	364	10/10	239	0.02
gr202	202	3,250	10/10	1,463	8.45
gr21	21	495	10/10	370	0.02
gr229	229	11,066	10/10	6,896	26.99
gr24	24	185	10/10	164	0.03
gr431	431	11,066	10/10	3,942	168.64
gr48	48	584	10/10	559	0.11
gr666	666	15,882	10/10	8,187	261.60
gr96	96	4,971	10/10	4,817	0.87
hk48	48	1,397	10/10	1,098	0.12
kroA100	100	2,101	10/10	**2,101**	0.01
kroA150	150	2,153	10/10	**2,153**	0.01
kroA200	200	2,264	10/10	2,249	0.06
kroB100	100	2,110	10/10	1,942	2.15
kroB150	150	2,203	10/10	2,082	1.41
kroB200	200	2,118	10/10	2,107	0.08
kroC100	100	2,294	10/10	2,253	0.17
kroD100	100	2,134	10/10	2,069	0.40

Table 28 *Results for symmetric MSTSP instances, TSPLIB [8] (Part I).*

Name	Size	Upper bound	Success	MBLKH Best value	MBLKH Total time
kroE100	100	2,165	10/10	2,002	1.61
lin105	105	1,594	10/10	1,477	0.80
lin318	318	2,441	7/10	2,408	3.87
nrw1379	1,379	1,480	10/10	1,382	206.03
p654	654	3,157	10/10	**3,157**	0.11
pa561	561	94	8/10	84	14.47
pcb1173	1,173	1,656	10/10	1,636	16.41
pcb3038	3,038	2,417	4/10	2,412	90.18
pcb442	442	2,202	10/10	**2,202**	0.06
pla7397	7,397	491,806	10/10	**491,806**	33.59
pr1002	1,002	8,917	10/10	8,296	61.97
pr107	107	8,254	10/10	6,826	0.02
pr124	124	7,377	10/10	**7,377**	0.01
pr136	136	8,237	10/10	**8,237**	0.01
pr144	144	7,836	10/10	7,224	0.66
pr152	152	8,387	10/10	8,265	0.06
pr226	226	9,510	10/10	9,360	0.58
pr2392	2,392	8,668	10/10	**8,668**	1.50
pr264	264	6,626	10/10	6,569	4.80
pr299	299	3,480	10/10	**3,480**	0.04
pr439	439	6,502	10/10	3,909	88.54
pr76	76	11,607	10/10	9,214	0.47
rat195	195	152	10/10	**152**	0.01
rat575	575	268	10/10	**268**	0.09
rat783	783	313	10/10	**313**	0.18
rat99	99	111	10/10	**111**	0.01
rd100	100	672	10/10	**672**	0.01
rd400	400	698	10/10	**698**	0.05
rl11849	11,849	10,970	1/10	10,130	28,339.12
rl1304	1,304	10,690	3/10	9,849	272.61
rl1323	1,323	10,654	2/10	10,289	158.66
rl1889	1,889	10,774	10/10	10,709	11.01
rl5915	5,915	10,661	1/10	10,110	5,793.81
rl5934	5,934	10,776	1/10	9,725	7,874.85
si1032	1,032	429	10/10	**429**	0.24
si175	175	304	10/10	**304**	0.01
si535	535	396	9/10	282	28.56
st70	70	63	10/10	**63**	0.00
swiss42	42	156	10/10	129	0.08
ts225	500	8,485	10/10	**8,485**	0.02
tsp225	225	262	8/10	233	3.57
u1060	1,060	9,224	10/10	8,345	177.94
u1432	1,432	3,538	10/10	**3,500**	7.20
u159	159	3,406	10/10	**3,406**	0.01
u1817	1,817	1,568	10/10	1,557	26.42
u2152	2,152	1,568	10/10	1,563	1.94
u2319	2,319	3,466	10/10	3,421	6.94
u574	574	1,719	10/10	1,568	45.14
u724	724	1,561	10/10	1,561	0.14
ulysses16	16	941	10/10	677	0.03
ulysses22	22	941	10/10	687	0.04
usa13509	13,509	247,403	1/10	171,745	261,458.72
vm1084	1,084	10,337	4/10	10,337	7.05
vm1748	1,748	10,896	10/10	**10,896**	0.74

Table 29 *Results for symmetric MSTSP instances, TSPLIB [8] (Part II).*

6. Conclusion

This paper has evaluated the performance of LKH on the BTSP and the MSTSP. Extensive tests have shown that the performance is quite impressive. For all BTSP instances either the best known solution were found or improved, and it was possible to find optimum solutions for a series of large-scale instances with up to one million vertices.

The developed software is free of charge for academic and non-commercial use and can be downloaded in source code via http://www.ruc.dk/~keld/research/BLKH/.

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