Jens Høyrup

Lengths, Widths, Surfaces

A Portrait of Old Babylonian Algebra and Its Kin

With 89 Illustrations

Springer

Jens Høyrup
Section for Philosophy and Science Studies
University of Roskilde
P.O. Box 260
DK-4000 Roskilde
Denmark
jensh@ruc.dk

Sources and Studies Editor:
Gerald J. Toomer
2800 South Ocean Boulevard, 21F
Boca Raton, FL 33432
USA

Library of Congress Cataloging-in-Publication Data
Høyrup, Jens.
Lengths, widths, surfaces : a portrait of old Babylonian algebra and its kin / Jens Høyrup.
p. cm. — (Studies and sources in the history of mathematics and physical sciences)
Includes bibliographical references and index.
ISBN 0-387-95303-5 (alk. paper)
1. Mathematics, Babylonian. 2. Algebra. I. Title. II. Sources and studies in the history of mathematics and physical sciences.
QA22 .H83 2001
510.935—dc21 2001032839

Printed on acid-free paper.

Production managed by Timothy Taylor; manufacturing supervised by Erica Bresler.
Photocomposed copy prepared from the author's files.
Printed and bound by Maple-Vail Book Manufacturing Group, York, PA.
Printed in the United States of America.

9 8 7 6 5 4 3 2 1

ISBN 0-387-95303-5 SPIN 10838641

Springer-Verlag New York Berlin Heidelberg
A member of BertelsmannSpringer Science+Business Media GmbH

To all the Assyriologist-friends in Copenhagen,
Leningrad, Illinois, and Germany East and West
who never refused assistance;

to Peter, Jöran, and Jim;

and in memory of O. Neugebauer

ogni approfondimento di ricerca rivela una complessità di elementi dei quali precedenti sintesi non avevan tenuto sufficiente conto, e, se li avevano presi in considerazione, non era sembrato che infirmassero una tesi di primaria importanza, di solito condizionata dal gusto imperante al tempo in cui venne formulata quella sintesi.

Mario Praz, *Gusto neoclassico*

Preface

"[...] it is through wonder that men now begin and originally began to philosophize" – thus Aristotle's *Metaphysica* 982[b]12 [trans. Tredennick 1933: I, 13]. Some 25 years ago *I* started wondering when reading the secondary literature about the early history of mathematics: what could be the reasons that induced the Babylonians to work on second-degree equations, as it was said they did? Obviously not practical applicability – nor, however, it appeared, that kind of curiosity which made the ancient Greeks create mathematical *theory*.

In parallel with many other questions, I pursued the matter until I believed – around 1980 – to have arrived at least at a rough explanation. Looking at what I wrote back then I can still recognize the inception of my present ideas about the historical sociology of Babylonian mathematical knowledge; but beyond the general Assyriological literature, my basis consisted of translated sources whose interpretation had been commonly accepted since the 1930s.

In 1982 I gave a guest lecture in Berlin on my sociological interpretation, after which a member of the audience asked me *what* this Babylonian algebra looked like. I answered in agreement with what I had understood on the basis of the translations, and thus gave a picture close to the rhetorical algebra of the Middle Ages. Peter Damerow, who had organized the session, at that moment asked me why I was so sure, and showed a geometrical interpretation which Evert Bruins had proposed for a particular text; I recognized the diagram from one of the geometrical proofs from al-Khwārizmī's *Algebra* (it is shown below in Figure 88, p. 413), which made me curious. I got hold of a grammar and a dictionary and soon realized that the diagram was totally irrelevant in the context where Bruins had used it; but I also discovered that the current interpretation of the Babylonian "algebraic" texts was made to fit the numbers but did not agree with what followed from a careful reading of the words between the numbers.

For the outsider, Assyriology comes close to being an occult science, and it took some years before I was able to publish a decent detailed account of my arguments and my results [Høyrup 1990]. That I got so far was largely due to the support I got from Bendt Alster, Mogens Trolle Larsen, and Aage Westenholz of the Carsten Niebuhr Istitute, University of Copenhagen, and to the discussions I had with the participants in the "Workshops on Concept Development in Babylonian Mathematics" organized in Berlin in 1983, 1984, 1985, and 1988 – especially with Peter Damerow, Robert Englund, Jöran Friberg, Hans Nissen, Marvin Powell, Johannes Renger, and Jim Ritter. I also got precious advice and patient encouragement from Wolfram von Soden, even though it took me years to convince him that I might be on the right track.

That I got further is thanks to the colleagues who prevented me from concentrating all my scholarly energies on Mesopotamia, and seduced me into pursuing parallel work on ancient Greek, Islamic, and Latin medieval mathematics. Though I started in the likeness of Columbus, hitting land on a course I had initially chosen for the wrong reasons, I continued rather like Odysseus, visiting many unfamiliar countries, staying long with Circe and with Calypso. I also lost some experienced companions and masters on the way whom I think of with much regret – first Kilian Butz and Kurt Vogel, more recently Wolfram von Soden and Wilbur Knorr; I even visited the realm of the dead and learned immensely from the shadows of Thureau-Dangin and Neugebauer.

I was never left alone on the shore of Ithaca (if that is where I am now), but the possessions on my shelves are no less precious for me than the gifts of the Phaeacians for Odysseus: books, articles, letters from colleagues, and my own notes and writings on many intersecting themes. The pages that follow build on these riches, synthesized as far as I can at the present stage. The core of the argument is an analysis of the techniques and conceptualizations of Babylonian "algebraic" and related mathematics from the "Old Babylonian" earlier second millennium BCE (the "golden age" of Babylonian mathematics), based on texts in transliteration and "conformal translation"; on this foundation, a global portrait of the mathematical type in question is delineated.

These are the topics of Chapters I–VII. They deal with a moment in the history of mathematics, but the approach is not historical: it is synchronous and does not ask about the development nor, *a fortiori*, about the forces that shaped this development. The rest of the book (Chapters VIII–XI) is devoted to history proper: the historical shaping of Old Babylonian mathematics itself, the detailed geographical and chronological pattern; the origins and transformation; and, finally, kinship and historical influence.

I shall abstain from reformulating in prose what may just as well be read from the table of contents, and close this *prolegomenon* with three technical remarks:

All translations into English in the following – both from the sources and from modern publications – are mine, if no other translator is identified.

References mainly follow the author/editor-date system (with alphabetization after first author in the bibliography, pp. 418*ff*). However, standard editions of Babylonian texts and Assyriological reference works are referred to by the customary abbreviations, which are also listed in the bibliography.

Babylonian tablets are referred to by habitual museum or publication numbers. The “Index of Tablets” (pp. 426*ff*) inventories all tablets referred to in the text and refers to the publications from which I have taken the single texts. It also lists the references to each text in the preceding pages.

Jöran Friberg read and commented valuably on part of the first draft and Eleanor Robson on the second version, for which I thank both sincerely. I hardly need to point out that I remain responsible for everything, both where I have followed their suggestions more or less faithfully and where I have decided differently.

Contents